



Modules

*Big-time-functional
programming*

15-150 M21

Lecture 0712
12 July 2021

0 Packaging Data Together

Examples

```
Int.toString : int -> string
Int.compare  : int * int -> order
Int.min      : int * int -> int
Int.max      : int * int -> int
Int.abs      : int -> int
```

```
String.concat : string list -> string
String.concatWith :
    string -> string list -> string
String.implode : char list -> string
String.explode : string -> char list
String.isPrefix : string -> string -> bool
String.compare : string * string -> order
```

Other examples

```
List.null : 'a list -> bool  
List.length : 'a list -> int  
List.nth : 'a list * int -> 'a  
List.rev : 'a list -> 'a list
```

```
ListPair.zip :  
  'a list * 'b list -> ('a * 'b) list  
ListPair.unzip :  
  ('a * 'b) list -> 'a list * 'b list
```

```
Fn.id : 'a -> 'a  
Fn.const : 'b -> 'a -> 'b  
Fn.curry : ('a * 'b -> 'c) -> 'a -> 'b -> 'c  
Fn.uncurry : ('a -> 'b -> 'c) -> 'a * 'b -> 'c
```

0712.0 (Crypto.sml)

```
3 structure Foo =  
4   struct  
5     datatype blah = A of int | B of string  
6     val k = 3  
7     exception Badness  
8     fun g 0 = 1  
9       | g n = 2 + g(n-1)  
10    end
```

```
structure Int = struct ... end  
  
structure Bool = struct ... end  
  
structure String = struct ... end  
  
structure List = struct ... end  
  
structure Fn = struct ... end
```

1 Signatures and Transparent Ascription

Today's Slogan:

**There are some things the user
shouldn't know**

Example: Crypto structure

```
(* highly-secure pseudorandom number generator
   *)
```

```
Crypto.prng : int -> int
```

0712.1 (Crypto.sml)

```
14 structure Crypto =
15 struct
16   fun calc1 n = 1013 + (389 * n)
17   fun calc2 n = n mod 1039
18   val prng = calc2 o calc1
19 end
```

0712.2 (Crypto.sml)

```
23 signature CRYPTO =  
24 sig  
25   val prng : int -> int  
26 end
```

0712.3 (Crypto.sml)

```
30 structure Crypto2 : CRYPTO =  
31 struct  
32   fun calc1 n = 1013 + (389 * n)  
33   fun calc2 n = n mod 1039  
34   val prng = calc2 o calc1  
end
```

Can go in a signature:

- Types
- Datatypes
- Values
- Exceptions
- Structures

Notes:

- `type` declarations in the signature can be fulfilled by `datatypes` in the structure.
- If the `datatype` is given in the signature, it must be copied identically in the structure.
- `types` can also be given *concretely* or *abstractly* in the signature.

Concrete

```
signature SIGCo =  
  sig  
    type t = int  
  end
```

Abstract

```
signature SIGAb =  
  sig  
    type t  
  end
```

```
signature SIGAb = sig type t end
```

```
structure StructName1 : SIGAb = struct  
  type t = int  
end
```

Even if the type is left abstract in the signature, the user knows what it is implemented as.

```
structure StructName2 :> SIGAb = struct
  type t = int
end
```

If the type is left abstract in the signature, the user *has no idea* what it is implemented as. Remember to give them a method for making values of that type!

0712.4 (Crypto.sml)

```
62 signature PRINTABLE =
63   sig
64     type t
65     val toString : t -> string
66   end
67 structure IOP : PRINTABLE =
68   struct
69     type t = int option
70     fun toString NONE = "NONE"
71       | toString (SOME x) =
72         "SOME(" ^ (Int.toString x) ^ ")"
73   end
```

Check Your Understanding

Why do we want transparent in this case?

2 Opaque Ascription

Is there a type of natural numbers in SML?

0712.5 (Nat.sml)

```
3 fun fact (0 : nat) : nat = 1  
4   | fact n = n * fact(n-1)
```

```
type nat = int
```

0712.6 (Nat.sml)

```
8 signature NAT =  
9 sig  
10   type nat  
11   val zero : nat  
12   val succ : nat -> nat  
13   val abs : int -> nat  
14   val asInt : nat -> int  
15   exception Negative  
16   val ? : int -> nat  
17 end
```

0712.7 (Nat.sml)

```
21 structure Nat:>NAT =
22 struct
23   type nat = int
24   val zero = 0
25   fun succ x = x + 1
26   val abs = Int.abs
27   val asInt = Fn.id
28   exception Negative
29   fun ? n = if n<0 then raise Negative else n
30 end
```

0712.8 (Nat.sml)

```
34 fun smartfact (n : Nat.nat) : int =  
35   let  
36     fun fact 0 = 1  
37       | fact (k:int) : int = k * fact(k-1)  
38   in  
39     fact(Nat.asInt n)  
40   end
```

The `Nat` structure maintained the **invariant** that every integer which the user can obtain of type `Nat.nat` must be nonnegative.

By ascribing opaquely, the user isn't able to construct any values of the abstract type "on their own". So they must use the methods supplied by the structure, which can be guaranteed to maintain the invariant.

Example: Queues

Queues are a sequential, first-in first-out data structure.

We can start out with an empty queue

```
>
```

Then *insert* or *enqueue* an element

```
> 3
```

Then another

```
> 3 4
```

Then if we *dequeue*, it returns the first one we put in

```
> 4
```


0712.9 (QUEUE.sig)

```
2 signature QUEUE =  
3 sig  
4   type 'a queue  
5   val emp : 'a queue  
6   val ins : 'a * 'a queue -> 'a queue  
7   val rem : 'a queue -> ('a * 'a queue) option  
8 end
```

0712.10 (queue.sml)

```
2 structure LQ :> QUEUE =  
3 struct  
4     type 'a queue = 'a list  
5     val emp = []  
6     fun ins (n,l) = l @ [n]  
7     fun rem [] = NONE  
8       | rem (y::ys) = SOME (y, ys)  
9 end
```

Invariant $L : 'a \text{ LQ.queue}$ lists the elements in the queue from first-in to last-in.

```
13 structure LLQ :> QUEUE =  
14 struct  
15   type 'a queue = ('a list) * ('a list)  
16   val emp = ([], [])  
17   fun ins (n, (front, back)) = (front, n::back)  
18   fun rem ([], []) = NONE  
19     | rem (y::ys, back) = SOME (y, (ys, back))  
20     | rem ([], back) = rem (List.rev back, [])  
21 end
```

Invariant If $(f, b) : 'a \text{ LLQ.queue}$, then $f @ (\text{rev } b)$ lists the elements in the queue from first-in to last-in.

Outside scope of the course:

Amortized Analysis

Homework:

Prove that a user cannot tell the difference between LQ and LLQ

- Can use structures to package data together
- Can use signatures to provide interfaces for uses
- The modules system allows you to hide information behind abstraction boundaries

- Typeclasses and algebraic structures
- Functors
- Sets

Thank you!