Intro to Homotopy Type Theory

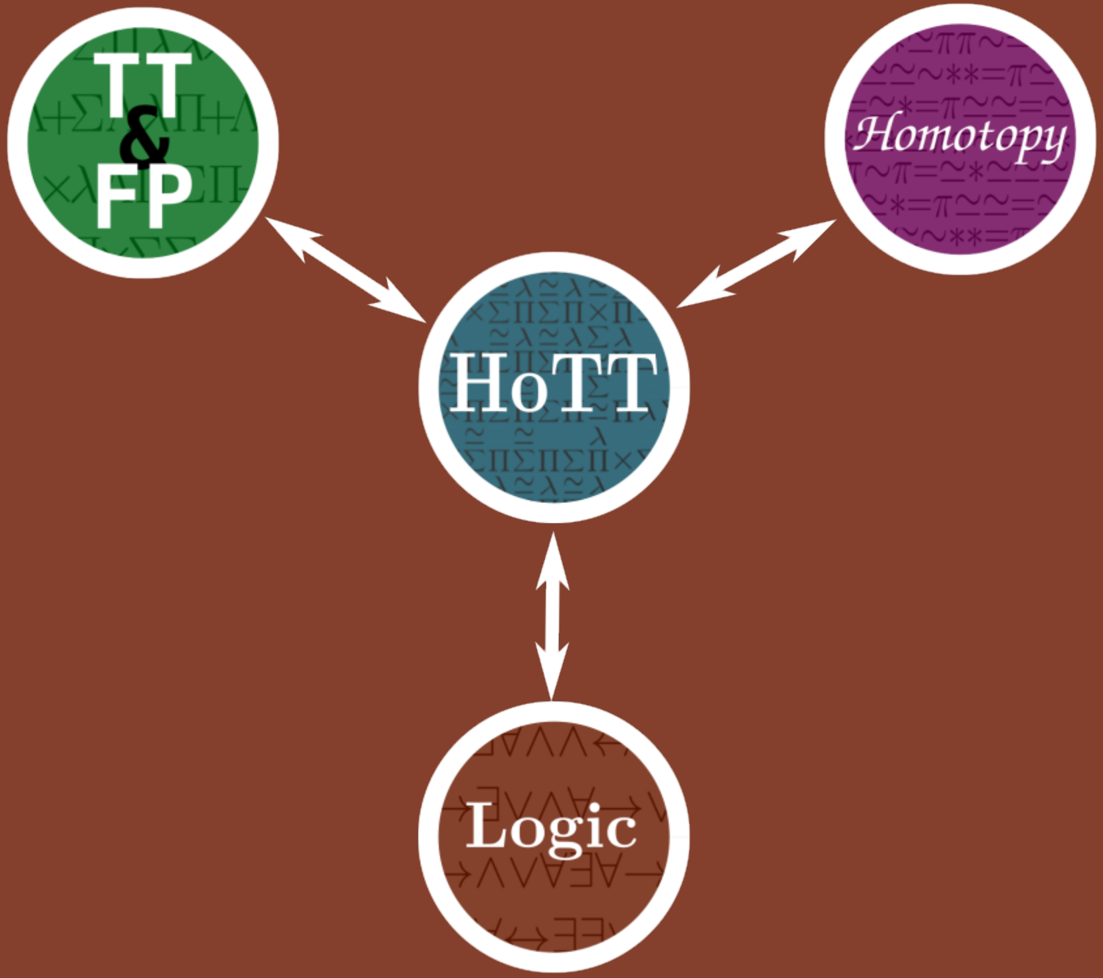
Three for One

Logic

Intro to Homotopy Type Theory, No. 1

Three for One

3 Logic


TT & FP
Homotopy
HoTT
Logic

- Theorems are types? Proofs are terms?
- ✓ What's homotopy theory? What's its connection to homotopy *type* theory?

Constructive vs. Nonconstructive

Example

Is there an $x > 2$ such that $2^x = x^2$?

- **Nonconstructive:** There must be, because...
- **Constructive:** Yeah, 4

Key Point: HoTT will be built around constructive logic

Logic Interpretation of HoTT:

Types are (constructive) propositions

Terms are proofs/witnesses

$t : T$ means t is a witness to the truth of T

Inhabitedness

"T is inhabited" $=$ there are terms of type T

"T is uninhabited" $=$ there are *no* terms of type T

(this is the constructive analogue of the distinction between "empty" and "nonempty")

Example: exp-square in Agda

Reminder for time-travellers:

	Written Prose	Agda
Judgmental Equality	$\equiv$	$=$
Propositional Equality	$=$	$\equiv$

There can be multiple distinct proofs/witnesses of the same proposition

(Note that constructive logic does not have to be proof relevant)

x such that $2^x = x^2$:

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w : ∃[ x ∈ ℕ ]  2 ^ x ≡ x ^ 2
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- $x = 4$

```
w = 4 , refl
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- $x = 2$

```
w = 2 , refl
```

multiple distinct proofs/witnesses of the same proposition

$\mathbf{1}$

- $\mathbf{1}$ represents a true proposition, because it's inhabited
- It's *uniquely* inhabited: there's exactly one term of type $\mathbf{1}$, i.e. exactly one witness of its truth
- We'll use $\mathbf{1}$/contractibility to capture the mathematical notion of "unique existence"