Groundwork to Higher Allegory Theory

24 Sept 2021 - CMU HoTT Seminar

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0 Relations and Correspondences

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This partial order is compatible with composition:

$$\begin{aligned} S \leq S' &\to (S \diamond R) \leq (S' \diamond R) \\ R \leq R' &\to (S \diamond R) \leq (S \diamond R') \end{aligned}$$

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Classical allegory theory focuses on binary meets on these posets:

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But for Rel, we can do meets (and joins) indexed over arbitrary sets:

$$\bigwedge_{i:I} R_i \equiv \lambda x.\lambda y.\prod_{i:I} R_i\ x\ y$$

$$\bigvee_{i:I} R_i \equiv \lambda x.\lambda y.\left\|\sum_{i:I} R_i\ x\ y\right\|$$

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and respects the poset structure:

$$R_1 \leq R_2 \ \leftrightarrow\ R_1^\dagger \leq R_2^\dagger.$$

Allegories are the abstract definition of this structure: a category enriched over posets with binary meets that is equipped with a involution operator, and satisfies certain laws.

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- $$\text{id}_X\ x\ x' \equiv (x = x')$$

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$$\mathsf{Corr}(R, R') \equiv (R \Rightarrow R') \equiv \prod_{x:X} \prod_{y:Y} (R\ x\ y) \to (R'\ x\ y)$$
- $$\mathsf{id}_R \equiv \lambda x.\lambda y.\lambda p.p$$

$$W \xrightarrow{R} X \xrightarrow{S} Y \xrightarrow{T} Z$$

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Whiskering

Given $R, R' : \mathrm{Corr}(X, Y)$, $\eta : R \Rightarrow R'$,
$S, S' : \mathrm{Corr}(Y, Z)$ and $\psi : S \Rightarrow S'$,

$$\begin{aligned}
\psi \triangleleft R \equiv\ & (\lambda x.\lambda z.\lambda(y, (p_{xRy}, p_{ySz})).(y, (p_{xRy}, \psi\ y\ z\ p_{ySz}))) \\
& : \prod_{x:X} \prod_{z:Z} ((S \circ R)\ x\ z) \to ((S' \circ R)\ x\ z) \\
& : (S \circ R) \Rightarrow (S' \circ R) \\
S \triangleright \eta \equiv\ & (\lambda x.\lambda z.\lambda(y, (p_{xRy}, p_{ySz})).(y, (\eta\ x\ y\ p_{xRy}, p_{ySz}))) \\
& : \prod_{x:X} \prod_{z:Z} ((S \circ R)\ x\ z) \to ((S \circ R')\ x\ z) \\
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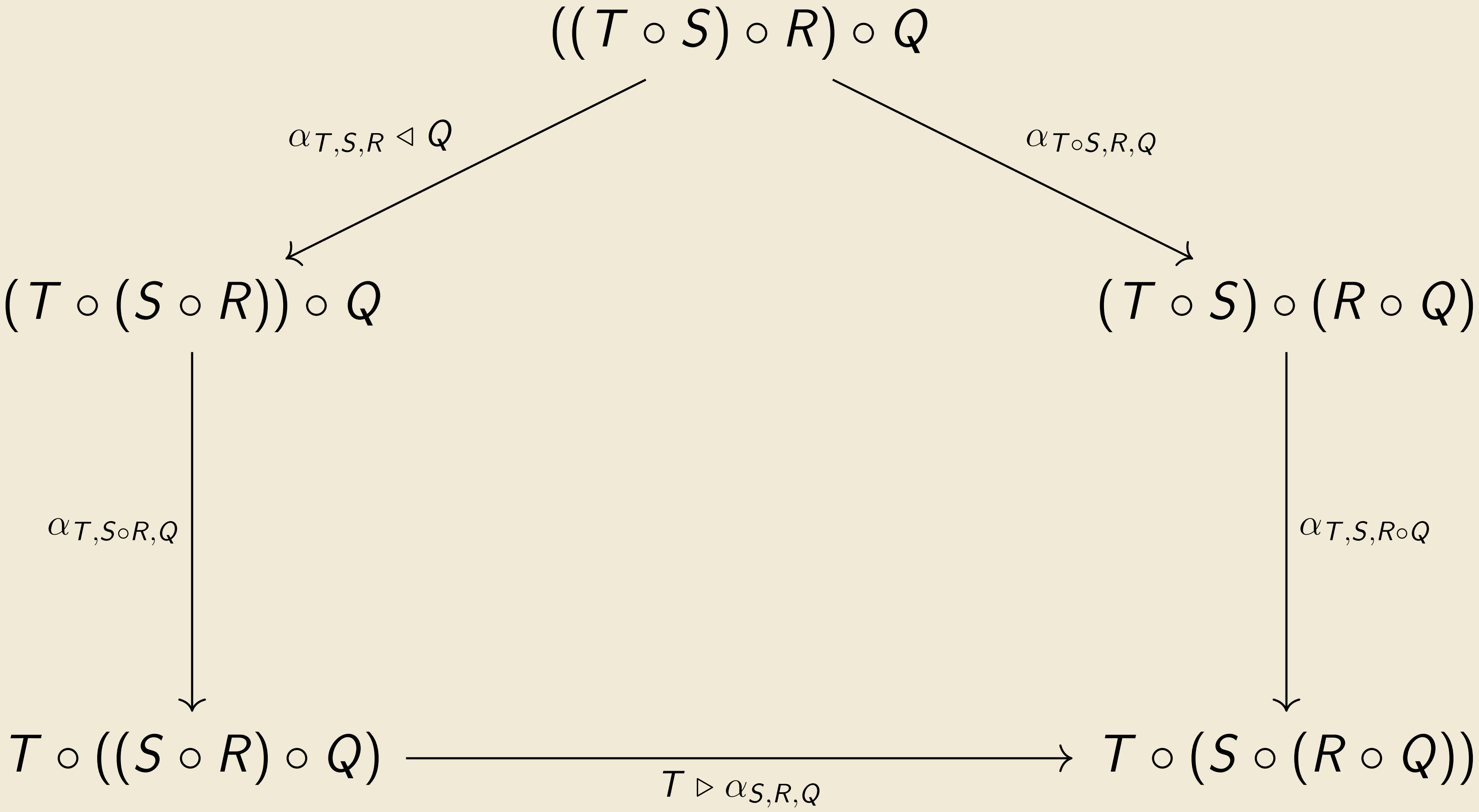
\end{aligned}$$


$((T \circ S) \circ R) \circ Q$
$\alpha_{T,S,R} \lhd Q$
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$T \rhd \alpha_{S,R,Q}$

Given any $v : V$, $z : Z$ and any

$$(w, (p_{vw}, (x, (p_{wx}, (y, (p_{xy}, p_{yz})))))) \; : \; (((T \circ S) \circ R) \circ Q) \; v \; z$$

,

$$\begin{aligned}
&\alpha_{T,S,R\circ Q}(\alpha_{T\circ S,R,Q}(w, (p_{vw}, (x, (p_{wx}, (y, (p_{xy}, p_{yz})))))))\\
&\equiv \alpha_{T,S,R\circ Q}(x, ((w, (p_{vw}, p_{wx})), (y, (p_{xy}, p_{yz}))))\\
&\equiv (y, ((x, ((w, (p_{vw}, p_{wx})), p_{xy})), p_{yz}))\\
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1 Maps and Equivalences

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Maps in allegory theory

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We can mimic this: given $R : \mathrm{Rel}(X, Y)$ and $x : X$, define

$$\mathrm{im}_R(x) \equiv \sum_{y:Y} R\ x\ y$$

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then a **map** is a relation with contractible images:

$$\mathrm{is\_map}(R) \equiv \prod_{x:X} \mathrm{is\_contr}(\mathrm{im}_R(x))$$

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$$(X \to Y) \simeq \sum_{R:\mathrm{Rel}(X,Y)} \mathrm{is\_map}(R)$$

Can we make something like this work for correspondences?

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Claim

$$\mathrm{is\_simple}(R) \;\leftrightarrow\; (R \diamond R^\dagger \leq \mathrm{id}_Y)$$

$$\mathrm{is\_entire}(R) \;\leftrightarrow\; (\mathrm{id}_X \leq R^\dagger \diamond R)$$

Idea: $\mathrm{Corr}(R \circ R^\dagger, \mathrm{id}_Y)$ and $\mathrm{Corr}(\mathrm{id}_X, R^\dagger \circ R)$ give us data about the simplicity and entirety of R

Claim

$$\left\|\mathrm{Corr}(R \circ R^{\dagger}, \mathrm{id}_Y)\right\| \times \left\|\mathrm{Corr}(\mathrm{id}_X, R^{\dagger} \circ R)\right\| \simeq \prod_{x:X} \mathrm{is\_contr}\left(\sum_{y:Y} \|R\ x\ y\|\right)$$

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Write $F \dashv G$ for the type of witnesses to this adjunction.

Claim For $R : \mathsf{Corr}(X, Y)$, the following are equivalent:

1. $\prod_{x:X} \mathsf{is\_contr}\left(\sum_{y:Y} \|R\ x\ y\|\right)$
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Conjecture We can drop some of the $\|-\|$'s.

Idea: Dualize all this with †

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3. $\left\|R^{\dagger} \dashv R\right\|$

3 Connections & Future Directions

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